Human minds crave narratives. We want to fit our experiences into a larger picture, and that larger picture into a broader framework, be that framework a scientific field, a culture, or a part of history. Mathematics is no exception, but I think the average student expects it to be. This, more than anything else, is what I set out to change in all of my teaching.

I have been teaching math in some capacity since high school– I was the teaching assistant to middle school precalculus courses, and have been tutoring a variety of students for around a decade. In undergrad I was the teaching assistant for several upper division math courses, and in graduate school I've acted as both teaching assistant and an instructor of record to undergraduate service courses in math. Beginning in my third year of graduate school, I have taught courses with no faculty oversight, and found the experience deeply motivating; I have spent the years since refining how I wanted to teach.

A common line I hear about teaching is 'teach with enthusiasm, and it will shine through to the students.' I take issue with this. Enthusiasm isn't enough– in fact, often, it is too much. A love of math is a beautiful thing, to be sure, but if it is not tempered by patience or awareness of your students' backgrounds, it can do more harm than good. Enthusiasm can be alienating, not inviting, especially for minority groups.

Many freshman in a first calculus course have been conditioned by years of social bias to believe that they will fail. They have been told for most of their lives that not only are they bad at math, but that there is nothing they can do to change that. This is a line surely every mathematician has heard– 'oh, math just wasn't for me.' No matter how genuine or well-intentioned, the enthusiasm of a young high-achieving mathematician will only reinforce that. So, what *can* change a mind convinced by years of ill-suited education that math just isn't for them? The answer I came to in my time as an instructor was to show them that mathematics, just like so many other fields, has rich and compelling stories to tell.

For example, when I taught linear algebra in Spring 2022 I decided to use nilpotent matrices as a running case study throughout the quarter, as they are useful for all manner of examples– non-invertible matrices, calculation of eigenvalues, the characteristic polynomial, conjugation to block form, etc. The general definition of nilpotence was not well-suited to the needs of the students, as the vast majority were in economics or computer science. This led me to simplify the definition and present them only with the 3×3 case.

On each problem set I walked them through a concept introduced that week in lecture using nilpotent matrices. Often, I'd structure the questions so they were easier with the last week's problem. This approach gave students a sense of comfort working with this string of problems– for all that the matrices were abstract, they were still a familiar type of matrix. The students knew the relevant properties, and had worked to understand them for weeks. Gradually, I had students start to ask more and more advanced questions about why we would consider such matrices. When did they appear? What could they tell us?

On the final, I posed the genuinely challenging question of fully classifying all 2×2 nilpotent matrices. Nearly every student solved it correctly, and more importantly– most were confident in their solution. This kind of confidence only comes from repeat exposure, and true comfort with the material and the end goal of a course.

It takes a lot to shift a mentality from "I cannot do this, I'm not good at math" to "I have an honest curiosity about this math object." In my opinion, it can only come from small positive reinforcement of their curiosity, and a patience to support students as they first learn to ask for the why.

If I tell a student a fact, they won't know where it fits in the subject, let alone why they should care. If I lead them through the discovery of a theme of facts, they won't just understand how it fits in, they will actively want to know more. This is true of the second group of students I'd like to discuss as well– even highly gifted students benefit from what they learn being woven into the

broader mathematical context. For these students, it doesn't just teach them to care about math, it teaches them how research math is actually done.

As mathematicians, we often discuss the motivating story of a piece of research, and incorporate it into how we present our research to others. This is a skill born of practice.

As another example, for several years I taught a particularly gifted 12 year old boy, and we worked through Rudin's *Principles of Mathematical Analysis* in some detail. Obviously, this student was very advanced. I discovered that he needed from me was not math, but the context that was missing from the math in the text. He wanted to know the story threading the theorems together, and how this book fit into the broader picture of modern research math. I found that providing this context didn't just help him learn, it motivated him to learn faster.

All a student needs to succeed is the motivation to try, and a teacher with the patience to support them as they do so. It is my opinion that nothing motivates the human mind more than the allure of an unfinished story.